Group Number: _____

FINAL REVIEW - TRUE, FALSE, OR NONSENSE?

For each statement below, decide as a group whether it is true, false, or nonsense (i.e. its truth cannot be evaluated, because some or all of the terms are used incorrectly). Write T, F, or N on the line. If a statement is true, explain briefly. If it is false or nonsense, make a slight adjustment to it so that it makes sense and is true. Turn in one copy per group.

(1) T The following system of linear equations is inconsistent:

$$\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 2x_2 = 4 \\ 3x_2 - 3x_3 = 3 \end{cases}$$

Row reduce.

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 2 & 0 & 4 \\ 0 & 3 & -3 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -2 & 2 & 4 \\ 0 & 3 & -3 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 9 \end{array}\right)$$

Inconsistent.

(2) N Given a subspace W of $\mathbb{R}^n$, $W^\perp$ spans the same basis as W.

A basis spans a subspace, not the other way around

"$W^\perp$ is not spanned by the same basis as W"

(3) T If A is an $n \times n$ matrix such that A^T is invertible, then the product of the eigenvalues of A is nonzero.

If A^T is invertible, A is invertible, so 0 is not an eigenvalue of A, so the product of the eigenvalues is nonzero.

(4) F Given three functions $a(t)$, $b(t)$, and $c(t)$, continuous and defined on $(-\infty, \infty)$, the differential equation $a(t)y' + b(t)y = c(t)$ has exactly one solution defined on $(-\infty, \infty)$.

$a(t)y' + b(t)y = c(t)$ has infinitely many solutions.

OR $\begin{cases} a(t)y' + b(t)y = c(t) \\ y(0) = 0 \end{cases}$ has exactly one solution (as an initial value problem)

(5) N If $\{\overline{v}_1, \ldots, \overline{v}_n\}$ is a basis for V, then $A = \begin{pmatrix} \overline{v}_1 & \cdots & \overline{v}_n \end{pmatrix}$ must be a square matrix.

Fix: replace V with $\mathbb{R}^n$.

If V is not a subspace of $\mathbb{R}^n$, this is not a matrix.

(6) N For any three vectors $\overline{u}$, $\overline{v}$, and $\overline{w}$ in $\mathbb{R}^n$, $(\overline{u} \cdot \overline{v}) \cdot \overline{w} = \overline{u} \cdot (\overline{v} \cdot \overline{w})$.

$(\overline{u} \cdot \overline{v})$ is a number, so it can't be dotted with w.

$\overline{u} \cdot (\overline{v} \cdot \overline{w})$ makes no sense for the same reason. Fix: $(\overline{u} + \overline{v}) + \overline{w} = \overline{u} + (\overline{v} + \overline{w})$.

(7) T If A is an $n \times n$ matrix, and $\{\overline{v}_1, \ldots, \overline{v}_n\}$ is a set of eigenvectors with distinct eigenvalues, then A is diagonalizable.

$\{\overline{v}_1, \ldots, \overline{v}_n\}$ must be linearly independent if their eigenvalues are distinct. So A is diagonalizable.

(8) F If A and B are $n \times n$ matrices, $|A + B| = |A| + |B|$.

Fix: $|AB| = |A||B|$.